CHAPTER FIVE

LOGARITHM

Introduction:

* In $\log_b N$, N is referred to as the number and b is referred to as the base.

* The logarithm of a positive number N to a given base b, is the power to which b must be raised so as to be equal to N.

* For example, if $\log_x y = k$, then $x^k = y$.

* If $\log_3 9 = 2 \Rightarrow 3^2 = 9$.

* Also, $\log_2 16 = 4 \Rightarrow 2^4 = 16$

* Since $\log_4 16 = 2$, then $4^2 = 16$

(Q1) Determine the value of x, given that

a) $\log_5 25 = x$ (b) $\log_2 4 = 4$

(c) $\log_2 32 = x$ (d) $\log_5 125 = x$

(e) $\log_5 625 = x$ (f) $\log_3 81 = x$.

<div align="center">Soln:</div>

(a) Since $\log_5 25 = x$, then $5^x = 25 \Rightarrow 5^x = 5^2$

$\Rightarrow x = 2$.

(b) if $\log_2 4 = x$, then $2^x = 4$

$\Rightarrow 2^x = 2^2 \Rightarrow x = 2$.

(c) Since $\log_2 32 = x$, then $2^x = 32$

$\Rightarrow 2^x = 2^5 \Rightarrow x = 5$.

(d) Since $\log_5 125 = x$, then $5^x = 125 \Rightarrow 5^x = 5^3 \Rightarrow x = 3$.

(e) If $\log_5 625 = x$, then $5^x = 625 \Rightarrow 5^x = 5^3 \Rightarrow x = 3$.

(Q2) Determine the value of y if

(a) $\log_y 4 = 2$ (b) $\log_y 16 = 2$

(c) $\log_y 36 = 2$ (d) $\log_y 81 = 4$

(e) $\log_y 64 = 3$ (f) $\log_y 27 = 3$.

<div align="center">Soln:</div>

Since $\log_y 4 = 2$, then $y^2 = 4$

$\Rightarrow y^2 = 2^2 = \Rightarrow y = 2$.

a). Since $\log_y 16 = 2$, then $y^2 = 16$

$\Rightarrow Y^2 = 4^2 \Rightarrow y = 4$.

a) Since $\log_y 36 = 2$, then $y^2 = 36$

$\Rightarrow y^2 = 6^2 \Rightarrow y = 6$.

(d) Since $\log_y 81 = 4$, then $y^4 = 81$

$\Rightarrow y^4 = 3^4 \Rightarrow y = 3$.

(e) Since $\log_y 64 = 3$, then $y^3 = 64$

$\Rightarrow y^3 = 4^3 \Rightarrow y = 4$.

(f) $\log_y 27 = 3$, then $y^3 = 27$

$\Rightarrow y^3 = 3^3 \Rightarrow y = 3$.

N/B: (1) If no base is written or indicated, then we are dealing in base 10.

* For example, $\log 10 = \log_{10} 10$ and $\log 8 = \log_{10} 8$.

* (2) If the value of the number and the base are the same, then the value of the log is 1.

* For example, $\log_{10}10 = 1$ and $\log_2 2 = 1$.

* Also $\log_5 5 = 1$ and $\log_4 4 = 1$.

Determination of values of logarithm:

*This can be done by either using a four figure table or a scientific calculator.

Using the four figure table:

*In this case, the decimal point must be after the first number.

* If this is not so, then it must be brought after the first number.

* If this point has to be moved or shifted once towards the left, then the character is 1.

* If it is moved twice, or by two steps, then the characteristic is two.

* If it is moved thrice or by three steps toward the left, then the characteristic becomes three and so on.

* If the decimal point is already after the first number, then there is no movement or shifting of this point, and the characteristic is zero.

(Q1) Determine the characteristic of each of these numbers:

(a) 2.45   (b)3.817

(c)24.5   (d)388.5

(e)24   (f)345

(g)2401   (h)73105

(i)4445.8 (j)3000.43

                        Soln:

(a) In 2.45, the characteristic is 0, since the point is already after the first number.

(b)Also in 3.817, the characteristic is zero.

(c) 24.5. In this case, the characteristic is 1, since the point has to be shifted one step to the left, in order to be after the first number.

(d) 388.5. In this case, the characteristic is 2, since the point must be shifted two steps left in order to appear after the first number.

(e)24 = 24.0. In this case, the characteristic is 1.

(f) 345 = 345.0. In this case, the characteristic is 2.

(g) 2401 = 2401.0. In this case, the characteristic is 3, since the point must be shifted three steps left, in order to appear after the first number.

(h) 73105 = 73105.0. In this case, the characteristic is 4.

(i) 4445.8. The characteristic is 3.

(j)3000.43. The characteristic is 3.

N/B:

- On the other hand, if the decimal point has to be moved once toward the right before it comes after the first number, then the characteristic is -1.

- If it is moved twice or two steps toward the right, then the characteristic is -2.

- If this movement towards the right is by three steps, then the characteristic is -3.

(Q2) Determine the characteristic of each of these numbers:

(a) 0.24   (b) 0.00789

(c) 0.0005   (c) 0.00085

Soln:

(a) 0.24. The characteristic is -1 in this case, since the point has to be moved one step to the right, in order to appear after the first number.

(b) 0.00789. The characteristic is - 3, since the point has to be shifted three steps to the right, in order to appear after the first number.

(c) 0.0005. The characteristic is - 4.

(d) 0.00085. The characteristic is - 4.
- In the determination of the values of the given logarithm using the table, the characteristic is first determined before the actual value of the log is determined from the table.

- In certain cases, what is referred to as differences may arise.

- The values of these differences which are found at the extreme right hand side of the table, must be added to the values of the logarithm to get our final value.

(Q3) Determine the value of each of the following:

(a) log 0.451   (b) log 0.2453

(c) log 0.245   (d) log 0.2453

(e) log 0.4569   (f) log 0.0171

(g) log 0.01719   (h) log 0.00028

(i) log 0.0002865 (j) log 0.03

(k) log 0.008   (l) log 0.0000895

(m) log 0.0000821368

Soln:

(a) In log 0.451, the characteristic is -1.

- We then determine the value of log 45 under 1, which is = 6542.

=> log 0.451 = -1.6542.

(b) In log 0.45, the characteristic is -1. Since log 0.45 = log 0.450, we determine the value of log 45 under 0 which = 6532.

The value of log 0.45 = - 1.6532.

(c) The characteristic of log 0.245 = -1.

Log 24 under 5 = 3892.

=> The value of log 0.245 = -1.3892.

(d) In Log 0.2453, the characteristic is -1.

- We then determine the value of log 24 under 5, and add to it the value of difference 3.

- Log 24 under 5 = 3892 and the value of the difference 3 in this case = 5.

3892 + 5 = 3897.

=> log 0.2453 = -1.3897.

(e) In log 0.4569, the characteristic = -1.

Log 45 under 6 = 6590 and the difference 9=9.

6590 + 9 = 6599.

Log 0.4569 =-1.6599.

(f) In log 0.0171, the characteristic = - 2.

Log 17 under 1 = 2330.

=> log 0.0171 = -2.2330.

(g) In log 0.01719, the characteristic = -2.

log 17 under 1 = 2330 and its difference 9 = 22.

2330 + 22 = 2352.

=> log 0.01719 = -2.2352.

(h) In log 0.00028 which is the same as log 0.000280, the characteristic = - 4 and log 28 under 0 = 4472.

=> log o.ooo28 = - 4.4472.

(i) In log 0.0002865, the characteristic = - 4.

Log 28 under 6 = 4564 and its difference 5 = 8.

4564 + 8 = 4572.

Log 0.0002865 = - 4.4572.

(j) log 0.03 = log 0.0300 and the characteristic = -2.

log 30 under 0 = 4771.

=> log 0.03 = -2.4771.

(k) log 0.008 = log 0.00800 and the characteristic= -3.

Log 80 under zero = 9031.

=> log 0.008 = -3.9031.

(l) In log 0.0000895, the characteristic = -5.

log 89 under 5 = 9518.

=> log 0.0000895 = -5.9518.

(m) In log 0.0000821368, the characteristic = -5.

Log 82 under 1 = 9145 and the difference 3 = 2.

9145 + 2 = 9147.

=> Log 0.0000821364 = -5.9147.

N/B: In log 0.000821368, we only consider log 82 under 1 difference 3.